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PARAMETER IDENTIFICATION AND ELASTIC-PLASTIC PROBLEMS: OPTIMIZATION STRATEGIES

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Abstract. Parameter identification is a technique which aims at determining model parameters based on a combination of experimental and numerical procedures. This work addresses identification of material parameters for elastic-plastic problems using optimization methods. Firstly, a comparative study is presented in which optimization techniques based on Genetic Algorithms, Univariate, Steepest Descent and BFGS methods are discussed. Identification of Carbon steel parameters based on tensile tests illustrates application of the BFGS, GA and GA-BFGS hybrid methods.

1 INTRODUCTION

In recent years, the development of robust computational models has made possible to efficiently simulate a wide range of metal forming processes. Such progress has been translated into the release of commercial packages able to simulate forming processes such as forging, rolling, deep drawing and extrusion amongst many others. However, the success of the simulations is directly linked to the capacity of constitutive models and respective parameters to accurately represent the experimental behaviour of the material. In general, industries have determined such parameters by means of mechanical tests described in technical standards using the assumption of homogeneous deformation. For instance, the material parameters obtained using tensile tests are determined for strains up to the necking onset, thereby severely limiting the level of plastic strain above which the results are no longer valid. Therefore, use of such material parameters in the simulation of metal forming operations involving large plastic strains would compromise the results.

The numerical-experimental technique known as parameter identification has emerged as the best approach to determine material parameters at large strains. The recent literature shows many works on the determination of constitutive parameters of elastic-plastic problems[1]. The investigation presents further studies on the application of optimization strategies to this class of problems. In a first part, a comparative study of the *Univariate* technique, *gradient-based* methods and *Genetic Algorithms* (GA) is discussed based on experimental data available in the literature. In a second part, a *hybrid strategy* combining *GA* with a *gradient-based* optimization method is assessed. The technique is illustrated for tensile tests of cylindrical specimens prepared according to the ASTM and NBR technical standards.

2 PARAMETER IDENTIFICATION AND THE OPTIMIZATION PROBLEM

Conceptually, parameter identification consists of finding a set of parameters, defined in this work as $\boldsymbol{p}$, which minimizes the difference between an experimental measure and a corresponding computed response with respect to a given norm. In this work, the computed response is obtained by solving the direct problem using a non-linear finite element approximation whereas the experimental response corresponds to measurement of selected variables of the experiment.

Parameter identification, therefore, can be formulated using optimization techniques, which consists of finding a minimum of the general problem

$$\begin{array}{lll} \text{Minimize} & g_0(\boldsymbol{p}) & \boldsymbol{p} \in R^n \\ \text{Subject to} & h_l(\boldsymbol{p}) = 0 & l = 1 \ldots m_h \\ & g_j(\boldsymbol{p}) \le 0 & j = 1 \ldots m_g \\ & p_i^{\inf} \le p_i \le p_i^{\sup} & i = 1 \ldots n \end{array} , \tag{1}$$

in which $\boldsymbol{p} \in R^n$ is the *design vector* (a vector containing *n design variables*), $p_i^{\inf}$, and, $p_i^{\sup}$, are upper and lower bounds of the design variables, respectively, $g_o(\boldsymbol{p})$ is the *objective function* to be minimized, subjected to *equality*, $h_l(\boldsymbol{p})$, and *inequality* $g_j(\boldsymbol{p})$ constraints.

The initial problem is complemented by the definition of the *objective function*, which in this work is based on the relative quadratic difference between experimental and computed response evaluated over the deformation process, so that

$$g_0(\boldsymbol{p}) = \sqrt{1/N \sum_{k=1}^{N} \left[\left(R_k^{MEF}(\boldsymbol{p}) - R_k^{Exp} \right) / R_k^{Exp} \right]}, \tag{2}$$

where R^{Exp} is the experimental response, $R^{MEF}(\boldsymbol{p})$ is the corresponding numerical response computed using a set of *design variables* $\boldsymbol{p}$, and N is the number of experimental points. In the present case, the measured and computed tensile loading are used to evaluate $g_o(\boldsymbol{p})$, and the hardening parameters correspond to $\boldsymbol{p}$. Therefore, in the identification process, the loading curve is computed using Finite Elements based on a given set of hardening parameters. The optimization process subsequently changes $\boldsymbol{p}$, so that, at the end of the identification procedure, g_o is minimum.

The literature shows several principles upon which optimization methods are based for multidimensional problems[2,3,4]. The most common approaches are *mathematical programming techniques, evolutionary methods* and, to a lesser extent, the *univariate approaches*. The *gradient-based algorithms* fall within the former, in which the gradient of $g_o(\boldsymbol{p})$ with respect to $\boldsymbol{p}$ is computed at each iterative step. *Gradient-based algorithms*, therefore, requires a continuous and twice differentiable objective function and constraints (the Hessian must be continuous). A disadvantage of such methods is the influence of the initial parameters on the process when the problem is noncovex. Moreover, the nonconvexity of parameter identification problems favours existence of multiple local minima.

Evolutionary methods consist of optimization algorithms based upon a generic *population* and use concepts inspired in biological mechanisms. The idea behind all variants of evolutionary algorithms is that each candidate plays the role of an *individual*, part of a *population*, and that some individuals are selected to generate the next *generations*. Selection and evolution of the population takes place by a recursive application of operators mimicking

biological evolution, such as *mutation* and *combination*. Besides no requirement of differentiability of the objective function and restrictions, the advantage of evolutionary algorithms is their theoretical capacity to determine the global minimum despite the existence of multiple local minima and plane regions (very small gradients). However, their obvious disadvantage lies on the high computational cost due to the large number operations required.

In addition to the *gradient-based* algorithms and *evolutionary* methods, the *univariate strategy* can also be used to multidimensional problems. In this case, a one-dimensional optimization technique is used to minimize one design variable at a time, seeking to produce a sequence of improved approximations to the minimum point.

2.1 Univariate methods

The concept of the method is described in Rao[3] for general optimization problems. In this method, one seeks to produce improved approximations to the minimum point by changing only one design variable at time and assuming that the remaining variables are constant during the process. The procedure is repeated successively for each design variable until a global convergence criterion is reached. This strategy requires application of a *one-dimensional* optimization method, giving rise to several possible combinations. In this work, this strategy is used is conjunction with the *Golden Section* one-dimensional strategy[2]. It is worthy to mention that the *Golden Section* method does not require computation of the gradient of the objective function. Convergence for the present implementation of the *Univariate* method is established by the mean quadratic relative difference of the current and previous set of material parameters, $\phi(\boldsymbol{p})$ (please, see Eq. (6)).

2.2 Gradient-based methods

Gradient-based methods are iterative in their essence, so that, at each iteration a new set of design variables are determined leading to minimization of the objective function. The iterative procedure is repeated until convergence is reached. The optimization literature indicates many mathematical strategies to account for the gradient in the iterative process. This work addresses the *Steepest Descent* and *modified Newton* with Hessian matrix computed using the *BFGS* equations.

For the sake of objectivity, this section presents a summary of the methods and the reader is referred to Arora[2] and Rao[3] for further insights on general purpose optimization methods. In this class of problems, the necessary conditions for a design vector $\boldsymbol{p}$ be a local minimum are established by the *Karush–Kuhn–Tucker* conditions (or *KKT* conditions). Most parameter identification techniques constitute unconstrained problems and the KKT conditions require only a null gradient of the objective function at the optimum point. Therefore, in order to ensure that $\boldsymbol{p}^*$ is a local minimum, $g_0(\boldsymbol{p})$ is expanded in a Taylor series in the neighbourhood of $\boldsymbol{p}^*$ so that

$$g_0(\boldsymbol{p}) - g_0(\boldsymbol{p}^*) = \frac{1}{2}\boldsymbol{d}^T\boldsymbol{Q}(\boldsymbol{p}^*)\boldsymbol{d}\,, \quad \text{where} \quad \nabla g_0(\boldsymbol{p}^*) = \boldsymbol{0} \quad \text{and} \quad \boldsymbol{d} = \boldsymbol{p} - \boldsymbol{p}^*\,, \tag{3}$$

in which $\boldsymbol{p}$ is sufficiently close to $\boldsymbol{p}^*$, $\boldsymbol{Q}$ is known as *Hessian* matrix and corresponds to the second derivative of the objective function with respect to the design variables, and $\boldsymbol{d}$ is the

search direction in the design space. Noticeably, since $g_0(\boldsymbol{p}) - g_0(\boldsymbol{p}^*) \geq 0$, the procedure leads to a minimum $\boldsymbol{p}^*$ only if the *Hessian* is positive definite. Therefore, Equation (3) suffices to ensure that $\boldsymbol{p}^*$ is a local minimum. Convergence for gradient-based optimization techniques are usually defined by the norm of the gradient of the objective function, $\phi(\boldsymbol{p}) = \| \nabla g_0(\boldsymbol{p}) \|$, however, in this work, some comparative examples adopt the same criterion established for the *Univariate* method (please, see section 3.1).

Steepest Descent method: The *Steepest Descent* method uses the negative of the gradient of the objective function as a search direction based on the fact that, in a given point $\boldsymbol{p}$, the direction opposite to $\nabla g_0(\boldsymbol{p})$ is the direction of fastest decrease of the objective function[2]. The optimization process is iterative so that the design variables at iteration k+1 are computed using the gradient of the objective function and the optimum step length, α. In the present case, a normalized search direction, $\boldsymbol{d}$, is used, so that

$$\boldsymbol{p}^{(k+1)} = \boldsymbol{p}^{(k)} + \alpha^{(k)} \boldsymbol{d}^{(k)} \qquad \text{and} \qquad \boldsymbol{d}^{(k)} = -\nabla g_0(\boldsymbol{p}^{(k)}) / \| \nabla g_0(\boldsymbol{p}^{(k)}) \| . \tag{4}$$

Computation of the gradient of the objective function, $\nabla g_o(\boldsymbol{p})$, known as sensitivity analysis, can be accomplished by using analytical, semi-analytical or numerical strategies. In this work $\nabla g_o(\boldsymbol{p})$ is computed using central finite differences. The optimal step size, α, represents the scaling along the search direction, being computed by solving a one-dimensional minimization problem and assuming that $\nabla g_o(\boldsymbol{p})$ is orthogonal to the search direction. In the present implementation, the *Golden Section* method is utilised to calculate α along the search direction.

Newton's method: The classical Newton's method is derived from the second-order expansion of the objective function in a Taylor series by assuming that the gradient of the objective function is null for iteration k+1. Contrasting to the *Steepest Descent* algorithm, Newton's method presents a quadratic rate of convergence in the vicinity of the optimal point. However, to ensure convergence under such condition, the Hessian must remain positive definite and computation of the optimum step size must be included. Therefore, the *modified Newton's method* using a normalized search direction can be represented as

$$\boldsymbol{p}^{(k+1)} = \boldsymbol{p}^{(k)} - \alpha^{(k)} \boldsymbol{Q}^{(k)^{-1}} \nabla g_0(\boldsymbol{p}^{(k)}) / \| \nabla g_0(\boldsymbol{p}^{(k)}) \| = \boldsymbol{p}^{(k)} + \alpha^{(k)} \boldsymbol{d}^{(k)} . \tag{5}$$

An evident drawback of the *modified Newton's method* is the requirement to calculate the Hessian matrix at each iteration. The search for alternative approaches to evaluate the Hessian gave rise to the so-called *Quasi-Newton* methods, which use approximations of $\boldsymbol{Q}^{-1}$ computed from the gradient of the objective function. In this work, the strategy proposed by *Broyden, Fletcher, Goldfarb and Shanno*[2] (*BFGS*) was used to calculate the inverse of the Hessian.

2.3 Evolutionary methods

Evolutionary methods are heuristic search strategies inspired in natural phenomena and biological mechanisms. One of the most widely used methods is the *Genetic Algorithm* (*GA*), which attempts to mimic natural evolution of a generic population[4]. In such techniques, the optimization process evaluates only $g_o(\boldsymbol{p})$ and does not require a continuous and differentiable

objective function. When convexity can not be ensured in advance, as most parameter identification problems, *GA* methods yield good approximation to obtaining the global minimum within the design space. In addition, the algorithm can be easily parallelized, making possible to use multiprocessor and distributed computing. It is relevant to mention that *GA*s have been used in areas as widely different as economics, task scheduling, Computer Aided Design (CAD), state assign problems, robot trajectory generation, routing in telecommunications network and many others. Application of *GA* to parameter identification is a relatively new endeavour, especially when addressing finite strain elastic-plastic problems. The recent works of Chaparro et al.[5], Munõz-Rojas et al.[1,6] and Aguir et al.[7] illustrate application of *GA* to this class of problems.

Genetic Algorithms account for two fundamental steps: *selection* and *reproduction*. The former is the process of choosing parents for reproduction, whereas the latter creates offspring from one or two parents. Initially, the *initial population*, *i.e.* a set of design vectors containing the material parameters $\{\boldsymbol{p}_1 , \boldsymbol{p}_2 , \ldots , \boldsymbol{p}_{ni}\}$, is randomly generated within the design space. In general, the literature designates each design vector, $\boldsymbol{p}_i$, as an *individual* (or *phenotype*). The number of individuals of a population, *ni*, is a variable of the method defined *a priori*. The most usual structure of *GA* encodes the design vector (or an *individual*) in a binary string upon which the genetic operations can be easily applied. The *gene* of an *individual* corresponds to a single material parameter, and is represented by a number of binary units (1 or 0) – *number of bits*. The accuracy of the search is determined by the number of bits used to encode a *gene* (or a single material parameter).

Generation of the initial *population* is followed by computation of the objective function for every *individual* and application of the *selection*. The *fitness proportionate selection*, also known as *roulette-wheel* selection, was used in this work to select parents. This method assigns a proportion of the roulette wheel according to the value of the objective function, *i.e.* smaller objective functions yield larger proportions, thereby increasing the probability of selection after a random rotation of the roulette wheel.

The formation of a new *generation* is completed by application of the genetic operators of *combination* and *mutation* to the selected individuals. In the process of *combination,* two new *individuals* (referred as *offspring*) are generated from a random combination of genes of pre-selected parents. The *mutation* operation randomly alters the values of genes of the individual according to a given rate. *Mutation* prevents the *GA* to reach an early convergence to a local minimum. The processes of selection and reproduction are subsequently applied to form new generations until a convergence or stopping criterion is reached.

2.4 GA-BFGS hybrid method

Hybrid methods can be generally defined as a combination of two or more optimization methods. This strategy is mainly used to improve the accuracy of results and convergence of the optimization process. The literature shows just a few works devoted to application of hybrid approaches to parameter identification of elastic-plastic problems. Ponthot and Kleinermann[8] investigated application of the *conjugate gradient*, *BFGS*, a modified *Globally Convergent Method of Moving Asymptotes*, *Levenberg-Marquardt* and *Gauss-Newton* optimization methods to identification of hardening parameters of a von Mises material. The

authors discussed also several possible combinations of the aforementioned methods aiming at avoiding local minima.

Hybrid approaches combining *Genetic Algorithms* and *gradient-based* optimization procedures were proposed by Chaparro *et al.*[5] and Muñoz-Rojas *et al.*[1,6]. Chaparro *et al.*[5] combine a *Genetic Algorithm* and the *Levenberg-Marquardt* method to determine hardening parameters of anisotropic materials. Muñoz-Rojas *et al.*[1,6], aiming at the GTN damage model, proposed a combination of *GA* with either *Sequential Linear Programming* or *Globally Convergent Method of Moving Asymptotes* methods. In both works the *Genetic Algorithm* is used with the objective of reducing the design space of the gradient based method by providing initial parameters closer to the global minimum. A hybrid identification procedure using *artificial neural networks* was developed by Aguir *et al.*[7] as an alternative to the finite element calculations to evaluate the objective functions within the *Genetic Algorithm*. The authors used also a multi-objective strategy to account for experimental results for uniaxial and biaxial tensile tests.

In addition to the positive characteristics highlighted is section 2.3, *GA* can be used to assess the design space, making possible to define new lateral restrictions of the search region. On the other hand, stringent convergence requirements, large initial population, generations and number of bits demand higher computing resources and processing time. Therefore, aiming at improving the efficiency of the optimization process, a hybrid strategy combining *GA* and the *modified Newton – BFGS* gradient-based method is proposed. The procedure can be described as follows: (*i*) Initially, the *Genetic Algorithm* is applied seeking to reduce the search region of the gradient-based method (*i.e.* improve estimation of the initial parameters). This strategy intends to circumvent the well-known convergence problems and convergence to local minima associated with defining initial parameters in gradient-based methods. There are no established rules on defining the best initial population size or other *GA*-related parameters. (*ii*) The second step consists of using the *modified Newton – BFGS* method with initial parameters, $\boldsymbol{p}^{(0)}$, given by the best individual (*i.e.* smallest value of the objective junction) provided by the last generation of the GA. This strategy has proven to be robust and accurate since the *modified Newton – BFGS* method presents high convergence rate in the neighbourhood of the optimal point.

3 NUMERICAL EXAMPLES AND DISCUSSIONS

The parameter identification techniques, including the convergence process, depend upon (*i*) the finite element approximation, (*ii*) the optimization method, (*iii*) the finite element mesh (*iv*) the number of increments of the non-liner mechanical solution, (*v*) the initial parameters, and (*vi*) the convergence or stopping criteria. The following sections address some of the aforementioned aspects for the identification techniques summarized in section 2. The first example presents an assessment of the identification strategies (*i*) *Univariate – Golden section*, (*ii*) *Gradient-based – Steepest Descent*, (*iii*) *Gradient-based – modified Newton-BFGS* and (*iv*) *Genetic Algorithm,* using experimental data available in the literature. Emphasis is placed on the *BFGS* method, for which effects of the number of mechanical increments and mesh size is discussed. The second example shows application of the *modified Newton – BFGS* method and *hybrid GA-BFGS* strategy to determining constitutive parameters

based on uniaxial tensile tests. The effects of the specimen geometry and initial parameters are also investigated.

3.1 Assessment of individual techniques

The experimental load *vs.* displacement curve presented by Ponthot and Kleinermann[5], corresponding to a special steel used in piping manufacture for the nuclear industry (Steel A-533, Grade B, Class 1), is adopted as reference in this example. The initial radius and reference length of the specimen are $r = 6.413$ *mm* and $\ell_0 = 26.67$ *mm* ($2\,\ell_0 = 53.34$ *mm* is the gauge length), respectively. The problem is assumed isothermal and axisymetrical. The finite element mesh used in the simulations attempts to reproduce reference [8] and contains 400 elements and 451 nodes with refinement at the necking region. The same yield stress curve was utilised in this example,

$$\sigma_Y = \sigma_0 + \zeta\varepsilon_p + (\sigma_\infty - \sigma_0)\left[1 - \exp\left(-\delta\varepsilon_p\right)\right] , \tag{6}$$

in which σ_∞, σ_0, ζ and δ are the parameters to be determined. The Young modulus and Poisson's ratio are assumed $E = 206.9$ *GPa* and $\nu = 0.29$, respectively. The initial set of material parameters for each method is presented in Table 1. Noticeably, the *Steepest Descent* and *BFGS* methods require only initial values, whereas the *Univariate* and *GA* techniques demand a search interval delimited by maximum and minimum values. The *GA* parameters used in this example are the following: population of 80 individuals, parameters encoded with 20 bits, 85 % of combination probability and 5 % of mutation.

Table 1. Initial and maximum/minimum values.

Method		σ_∞ [*MPa*]	σ_0 [*MPa*]	ζ [*MPa*]	δ [*m/m*]	Step size
BFGS Steepest Descent	Initial	650	500	325	20	0.5
Univariate – Golden Section	Maximum	800	600	400	30	-
Genetic Algorithm	Minimum	500	400	250	10	-

The perturbation adopted by the *Steepest Descent* and *BFGS* methods is defined by multiplying the initial parameters by a constant factor $f = 2\times10^{-6}$. The different nature of the optimization strategies used in this work recommends specific convergence or stopping criteria. However, in an attempt to harmonize the convergence assessment, a global quadratic measure of the relative uncertainty interval was used for the *Univariate*, *Steepest Descent* and *BFGS* methods. Notwithstanding, the characteristics of the *GA* prevent definition of similar stopping criterion. Therefore, in the present simulations, the stopping criterion for the *GA* is the difference of $g_o(\boldsymbol{p})$ computed for the worst and best individual. Thus

$$\textit{Univariate, Steepest Descent, BFGS}: \quad \phi(\boldsymbol{p}) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{p_i^{(k+1)} - p_i^{(k)}}{p_i^{(k+1)}}\right)^2} \le 10^{-3} , \tag{6}$$

$$\textit{Genetic Algorithm}: \quad \varepsilon_{GA} = \left| g_0^{(l)}(\boldsymbol{p}^{\text{worst}}) - g_0^{(l)}(\boldsymbol{p}^{\text{best}}) \right| \le 10^{-5} , \tag{7}$$

in which p_i is an individual parameter, n is the number of constitutive parameters and superscripts k and l indicate the iteration step and generation number, respectively. It is relevant to mention that assessment of the convergence for gradient-based methods can also be defined by using the norm of the gradient of the objective function.

Table 2. Final parameters and process convergence data.

Method	σ_∞ [MPa]	σ_0 [MPa]	ζ [MPa]	δ [m/m]	$g(p)$	CPU / CPU_{BFGS}
Genetic Algorithm	667.63	434.77	254.75	20.002	0.00956408498	21.52
Univariate – Golden Section	669.23	479.91	250.00	16.314	0.00986779019	17.39
Steepest Descent	668.02	450.60	252.53	18.764	0.00938025222	9.67
BFGS	678.19	471.25	218.13	15.524	0.00881813074	1
Ponthot and Kleinermann[8]	657.7	458.5	311.4	18.868	-	-

Table 2 presents the final parameter set, objective function and relative CPU time (with respect to the *BFGS* method) for each identification technique, whereas Figure 1 shows the corresponding loading curves. Ponthot and Kleinerman's[8] parameters are also indicated in Table 2. For the initial parameter set given in Table 1, the *BFGS* presented the best results owing to the smaller objective function (*i.e.* smaller relative errors between the experimental and numerical loading curves) and smaller processing time (*i.e.* higher convergence rate). It is relevant to mention that, despite its best performance, the *BFGS* method is highly sensitive to the initial parameters, as discussed in section 3.2. The initial *GA* internal parameters (*number of bits* and *population size*) lead to a relatively smaller objective function. However, the large number of evaluation of the objective function imposes a prohibitively high CPU time when using single-processor computing. In this example, the *Univariate* method required also high processing time without any significant gain in the objective function. The advantage of such method is its dependence on the maximum/minimum initial interval instead of a parameter set.

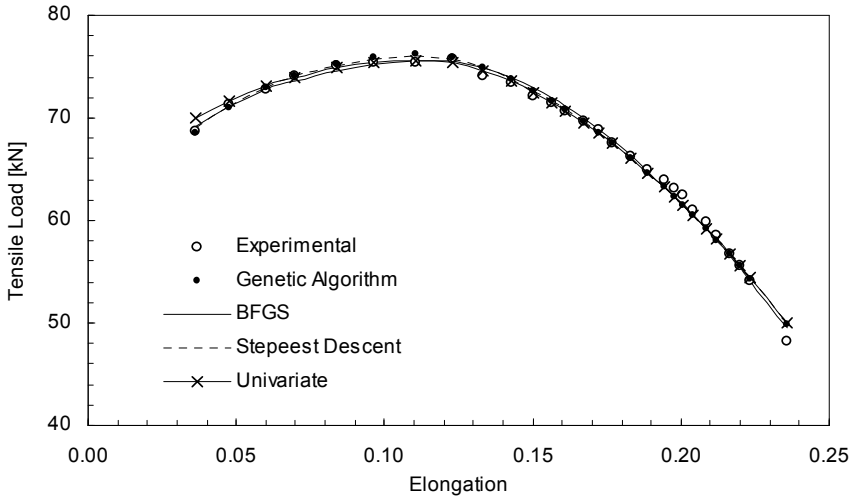

Figure 1: Loading curve.

3.2 Hybrid methods: Genetic Algorithm and BFGS technique

The best results provided by the *BFGS* optimization technique recommends further investigation on its application to parameter identification. However, the well-known dependence of gradient-based optimization methods on initial parameters and possibility of many local minima suggest use of alternative optimization techniques to estimate improved initial parameters for the *BFGS* method. This work uses a *Genetic Algorithm* in an attempt to determine initial parameters for the *BFGS* method closer to the global minimum. In this section, a brief assessment of the influence of the initial parameters in the identification process is presented, followed by the application of the *GA-BFGS* hybrid approach. Furthermore, the effects of the specimen geometry are also discussed.

Aiming at evaluating the effects of variations of geometry in the parameters, tensile tests were performed using specimens prepared according to the American ASTM E 8M-01 and Brazilian NBR ISO 6892 standards (referred in this work as ASTM and NBR, respectively). Figure 2(a) shows the ASTM and NBR specimens. The Brazilian NBR defines specimens with diameter and gauge length $d = 10 \pm 0.1\ mm$ and $\ell_0 = 70 \pm 0.15\ mm$, respectively, whereas the ASTM establishes $d = 12.5 \pm 0.2\ mm$ and $\ell_0 = 62.5 \pm 0.2\ mm$. Six NBR and ASTM specimens were prepared and tested, so that the corresponding median *tensile load x elongation* curves, shown in Figure 2(b), were used in the identification process.

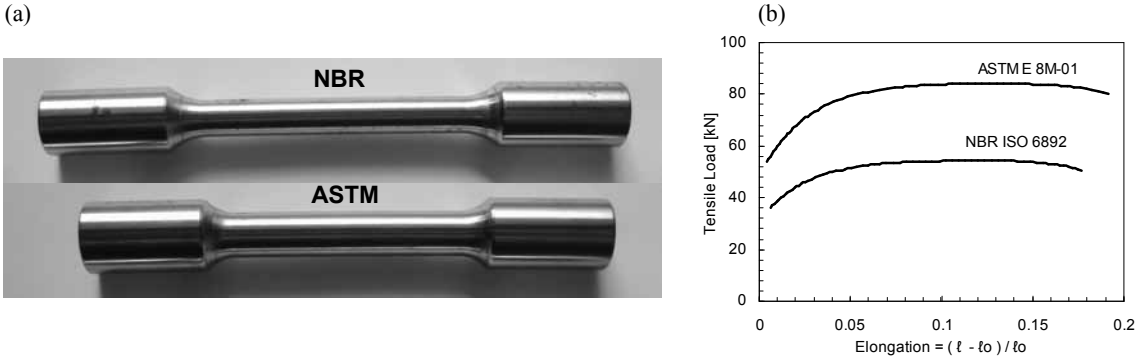

Figure 2: (a) Specimens prepared according to NBR and ASTM standards; (b) Tensile loading.

Table 3. Initial and maximum/minimum values.

Method		σ_∞ [*MPa*]	σ_0 [*MPa*]	ζ [*MPa*]	δ [m/m]	Step size
BFGS	Case (1)	1050	900	750	50	0.5
	Case (2)	700	600	500	30	
	Case (3)	250	210	180	10	
GA – BFGS Hybrid method	Maximum	1050	900	750	50	-
	Minimum	250	210	180	10	-

The geometrical models used in the simulations were defined according to actual measurements of the specimens. In this example radial symmetry and axisymetry were also assumed. A finite element mesh of 200 elements and 231 nodes, with progressive refinement at the centre region, was used for both NBR and ASTM specimens. The Young modulus and Poisson's ratio were $E = 200\ GPa$ and $\nu = 0.3$, respectively.

Table 3 shows the initial parameters used for the *BFGS* optimization strategy and the search region defined for the *GA-BFGS hybrid method*. The same set of initial parameters was used for identification based on NBR and ASTM specimens. The perturbation in this example was defined as in section 3.1. On the other hand, in this case, convergence for the *BFGS* was assessed by the norm of the gradient of the objective function, i.e. $\phi(\boldsymbol{p}) = \| \nabla g_0(\boldsymbol{p}) \| = 2 \times 10^{-6}$.

Results obtained by sole application of the *BFGS* method are presented in Table 4 for *Cases (1)* to *(3)*. One can observe that parameters obtained for *Case (2)* are close for both ASTM and NBR specimens. As well remarked in the literature, convergence for the *BFGS* optimization strategy is strongly dependent upon the initial parameters. No convergence was achieved in *Cases (1)* and *(3)* for the NBR and ASTM specimens within 50 iteration steps. Evolution of $\| \nabla g_0 \|$, shown in Figure 3, illustrates the convergence process for all cases.

Table 4. Parameters determined for ASTM and NBR specimens using the *BFGS* optimization method.

Method	Case	σ_∞ [*MPa*]	σ_0 [*MPa*]	ζ [*MPa*]	δ [*m/m*]	$g_0(p)$	CPU
ASTM	Case (1)	–	–	–	–	*No convergence*	2 h 11 m
	Case (2)	708.40	421.98	592.60	35.424	0.00266693349	38 m
	Case (3)	–	–	–	–	*No convergence*	1 h 52 m
NBR	Case (1)	–	–	–	–	*No convergence*	2 h 26 m
	Case (2)	720.65	426.61	552.82	35.042	0.00480087739	37 min
	Case (3)	–	–	–	–	*No convergence*	2 h 14 m

The *GA – BFGS* hybrid method is introduced as a possible solution for the convergence problem (owing to initial parameters) of gradient-based optimization strategies. Initially, in a first stage (A), the *Genetic Algorithm* is applied aiming at obtaining a point close to the global minimum (thereby avoiding local minima). The second stage (B) consists of application of the *BFGS* method to search for a minimum even closer to the global minimum.

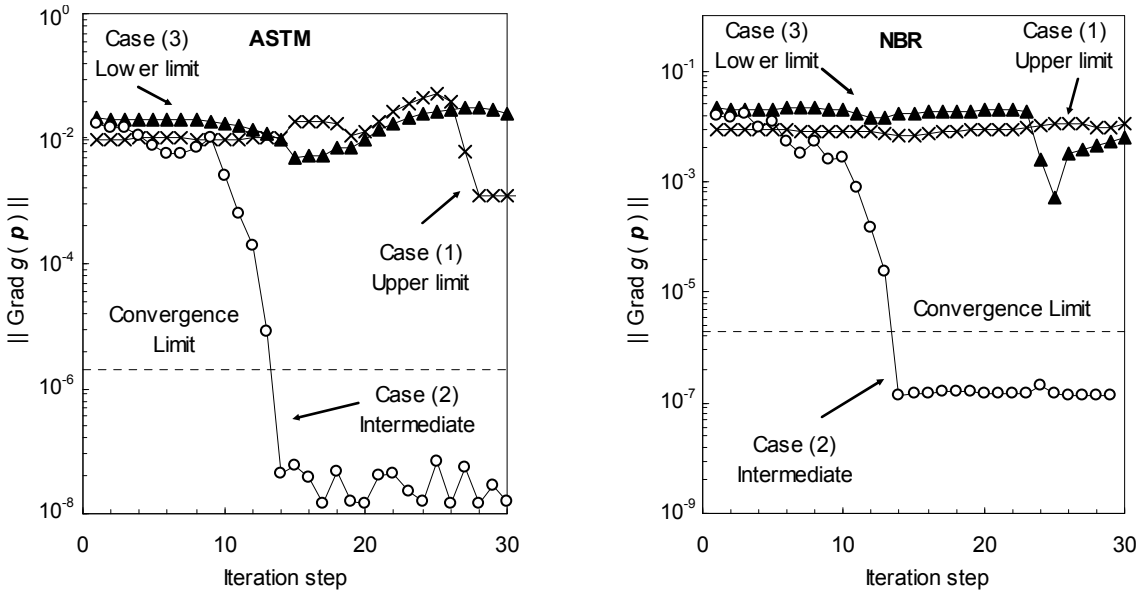

Figure 3: Evolution of $\|\nabla g_0\|$ for the initial set of parameters given in Table 3 for the BFGS method.

Table 5. Final parameters determined for ASTM and NBR specimens using the *GA – BFGS* hybrid approach.

Method	Stage	σ_∞ [*MPa*]	σ_0 [*MPa*]	ζ [*MPa*]	δ [m/m]	*g*(*p*)	CPU
ASTM	(A) *GA*	731.72	440.00	505.95	26.540	0.01200340080	1 h 26 m
	(B) *BFGS*	708.40	421.98	592.60	35.429	0.00266693349	32 m (*)
NBR	(A) *GA*	737.98	437.30	469.74	31.075	0.00802597455	1 h 57 m
	(B) *BFGS*	720.65	426.61	552.82	35.042	0.00480087739	30 (*)

(*) The total CPU time are: ASTM = 1 h 58 m and NBR = 2 h 27 m.

In this example, the parameters used for the *GA* are as follows: population of 60 individuals, parameters encoded with 10 bits, 85 % of combination probability and 5 % of mutation. It is interesting to note that smaller number of bits leads to "convergence" (difference between $g_0(\boldsymbol{p})$ of the worst and best individuals, ε_{GA}) at smaller number of generations, but with larger errors. In this case, larger errors are not relevant since the parameters obtained by applying the *GA* are used only as an initial approximation for the *BFGS* method. The stopping criterion used in this example is $\varepsilon_{GA} = 10^{-2}$.

Table 5 presents the parameter set obtained after stages (A) and (B) for ASTM and NBR specimens. The initial parameter set for the *BFGS* method, stage (B) of Table 5, were obtained after the *GA* reaches the stopping criterion. It is worthy to note that stricter stopping criteria would require larger number of generations, as indicated in Figure 4, without any improvement of the overall performance of the identification process.

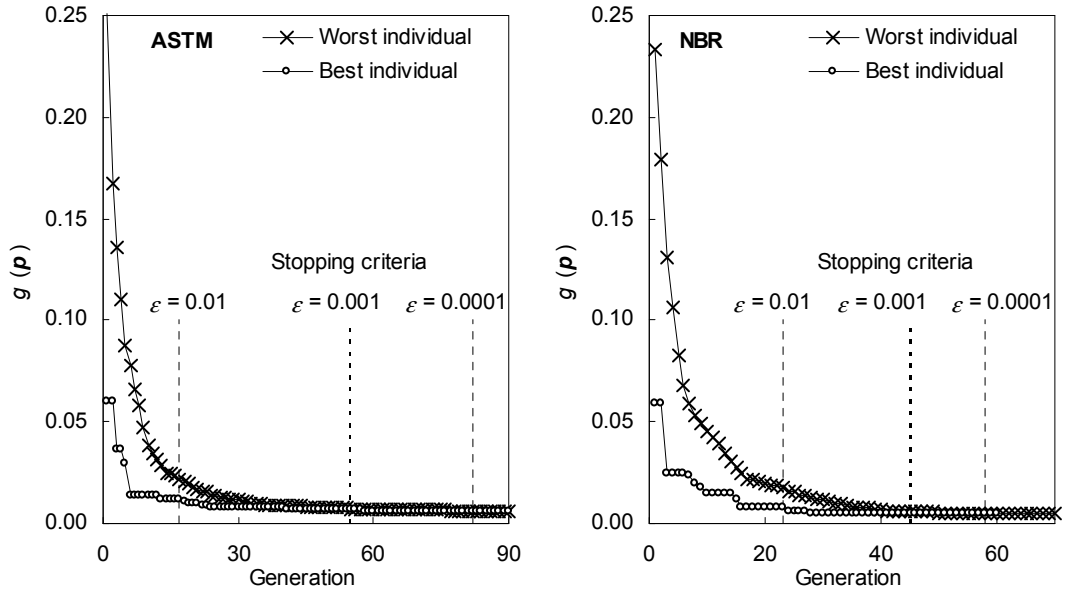

Figure 4: Evolution of the GA during Stage (A) and indication of the stopping creteria.

Case (2) of Table 4 (both ASTM and NBR) and Table 5 show that the *GA – BFGS hybrid method* yields the same parameters as the direct application of the *BFGS* technique. Therefore, it is possible to infer that such set of parameters indeed represent the global

minimum of the problem. Although the direct application of the *BFGS* method required less CPU time, its success is strongly dependent upon the initial parameters. On the other hand, in spite of requiring additional processing time, no convergence problems were observed when using the *GA – BFGS hybrid method*.

3 FINAL REMARKS

Parameter identification has become an essential task when developing new constitutive models. Direct measurement of constitutive parameters is not always possible thereby recommending use of inverse problem strategies, which in turn, are based upon optimization methods. In a first part, this work addressed identification procedures based on the *Univariate* approach, *Steepest Descent* and *BFGS* gradient-based methods and a *Genetic Algorithm*. A comparison of the aforementioned identification strategies shows that the *BFGS* method provided the best results (lower CPU time and objective function). However, this technique, as all gradient-based optimization methods, are strongly dependent upon the initial parameters and, therefore, liable to convergence problems, as discussed in a second example. Therefore, a *GA-BFGS* hybrid approach is proposed, i.e., the *GA* is applied first in order to estimate initial parameters for the *BFGS* method closer to the global minimum. The strategy was able to circumvent the convergence problems when attempting to determine material parameters of Carbon steel.

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